Analysis of Packet Transmission Delay Under the Proportional Fair Scheduling Policy*

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Abstract. It is expected that the proportional fair (PF) scheduler will be used widely in cdma2000 1xEV-DO systems because it maximizes the sum of each user's utility, which is given by the logarithm of its average throughput. In this paper, we address an influence of the PF scheduler on the packet transmission delay in base station (BS) and propose an analytic model.

1 Introduction

Recent advances in communication technology make appearance of the packet-based cellular systems such as cdma2000 1xEV-DO [1] and UMTS-HSDPA [2]. Being mainly targeted on high-speed data applications that are tolerant of some packet delay, it is reasonable that their schedulers focus on maximizing the sum of each user's utility. A good way of achieving it is to serve the users with good channel condition in order to utilize the time-varying feature of wireless channels. This approach increases the system throughput significantly. But, some users can be sacrificed since, in wireless environment, users have very different channel condition according to their location.

The proportional fair scheduler [3] is one of the most promising opportunistic schemes that balance system throughput and user fairness. It is very simple to implement, and also it is optimal in the sense of maximizing the sum of each user's utility that is given by the logarithm of average throughput for elastic traffic. However, owing to its reflection on channel state, the scheduler induces some variation on scheduling delay, and the variation may lead to unstable packet transmission delay. Since generally the delay variation makes negative influence on the performance of transport layer protocol and application, it is very important to have an accurate delay model that describes the delay variation. From that reason, we propose a packet transmission delay model in BS with PF scheduler in this paper. Also, we show the comparison of the analytic model and the simulation result using NS-2 [4].

* This work was supported by grant No.R01-2004-000-10588-0 from the Basic Research Program of the Korea Science & Engineering Foundation.

V.N. Alexandrov et al. (Eds.): ICCS 2006, Part IV, LNCS 3994, pp. 160–163, 2006.

2 Scheduling Delay Analysis

The wireless networks sometimes have a quite long delay because a base station may have many tasks to reduce the impact of the errors such as Forward Error Correction (FEC), interleaving, retransmission, and so on. In this section, we show an analytic model for BS delay, which is simplified with only the scheduling delay and the retransmission delay.

2.1 1 User Case

First, we build the model with 1 user. The user has the packet size of T bytes and is able to transmit X bytes whenever the scheduling slot is allocated. For example, if T is 1500 and X is 100 in constant, the time for servicing the packet is 15 slots. But when X changes depending on the channel state, the analysis comes to be more difficult.

For convenience of the analysis, we assume that X has an exponential distribution with average m (actually this assumption is exactly correct when the transmission rate linearly increases in proportion as SNR (Signal-to-Noise Ratio) on the Rayleigh channel). When we denote the data size that is successfully transmitted in flow i as X_i, the number of slots that is required to service the packet is $N(T)$. $N(T)$ is minimum N that satisfies $\sum_{i=1}^{N} X_i \geq T$. Analyzing this problem as the Poisson counting process, we can see that $N(T)-1$ has a Poisson distribution with both average and variance $\frac{T}{m}$.

We obtain the required number of slots to service a packet as above. However, owing to wireless channel error, the transmission does not always make a success even if BS successfully transmits the packet. In this model, we denote the error rate of each flow as p and assume that the error rate is independent of the transmission rate. At this time, to transmit the packet successfully in flow i, actually Y_i slots are taken. Because Y_i follows the discrete probability distribution with $Pr(Y_i = n) = p^{n-1}(1-p)$, we get $E(Y_i) = \frac{1}{1-p}$ and $Var(Y_i) = \frac{p}{(1-p)^2}$.

Actual number of slots to transmit a packet is given by $S = \sum_{i=1}^{N(T)} Y_i$, and we can obtain its average and variance as follows.

$$E(S) = E\{N(T)\}E(Y_i) = (\frac{T}{m} + 1)(1-p)^{-1}, \tag{1}$$

$$\begin{aligned} Var(S) &= E\{N(T)\}Var(Y_i) + E^2(Y_i)Var\{N(T)\} \\ &= (\tfrac{T}{m} + 1)p(1-p)^{-2} + (1-p)^{-2}(\tfrac{T}{m} + 1) \\ &= (\tfrac{T}{m} + 1)(1+p)(1-p)^{-2} \end{aligned} \tag{2}$$

2.2 *K* Users Case

Let's consider the case of K users. We assume that each user has a packet to transmit, and the packet size, T, and channel state are same in every user. Also assuming that the scheduler chooses a user and, only after transmitting the user's one packet, selects another user, we analyze the packet transmission time of the

Table 1. Configuration variables for simulation

Configuration Variables	Value
Schedule Interval (SCHED_INTERVAL_)	0.001667 (sec)
Transmission Power (Pt)	10 (Watt)
Bandwidth (BANDWIDTH)	1.25 (MHz)
Distance (DISTANCE)	100 (m)
Noise Density (ND)	2×10^{-14}

last-selected user. When the transmission time of $k-th$ selected user is denoted as D_k, our finding time is $D = D_1 + D_2 + ... + D_k = \sum_{k=1}^{K} D_k$. By applying Central limit theorem [5], we approximate D to a Gaussian distribution with the average $K \cdot E(D_k)$ and the variance $K \cdot Var(D_k)$.

$$\begin{aligned} E(D_k) &= E(S) = (\tfrac{T}{m}+1)(1-p)^{-1}, \\ Var(D_k) &= Var(S) = (\tfrac{T}{m}+1)(1+p)(1-p)^{-2}, \end{aligned} \tag{3}$$

Finally D follows the Gaussian distribution with the average $K(\frac{T}{m}+1)(1-p)^{-1}$ and the variance $K(\frac{T}{m}+1)(1+p)(1-p)^{-2}$.

For example, when we consider the case of T=1500, m=100, K=50, and p=0.1, the packet transmission time of the last selected user is as follows[1]. It is necessary to keep in mind that the inter-packet interval of a user comes from the scheduling delay.

- Constant rate with no channel error: 750 slots.
- Variable rate with no channel error: 800 slots with 50%.
- Variable rate with channel error, p: 889 slots with 50%.

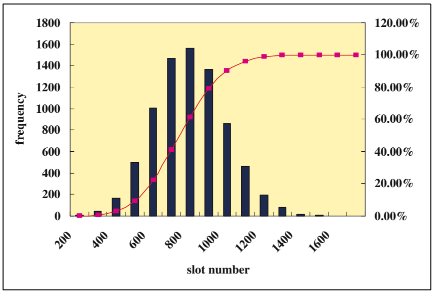
In this model, the scheduler services a user's packet sequentially but real PF scheduler services several users' packet little by little, depending on the channel state. Thus, every user finishes the packet transmission at similar time due to their mixed service time while the average rate of the allocated slots is so high as to have a better possibility that reduces the entire transmission time. Consequently every user has a similar finish time with the "last" user.

3 Simulation Result

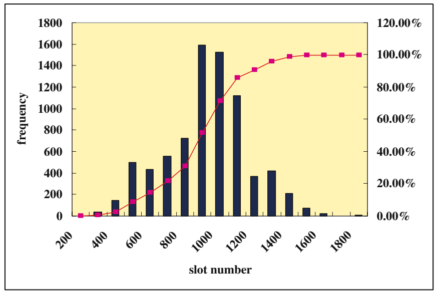
To validate the model, we perform simulation studies using NS-2. Some key configuration variables are summarized in Table 1.

The configuration values make average m=100, which is used in examples for our analysis. To remove the impact of transport layer protocol, we used UDP and CBR application. Simulation run time is 200 sec, and about 8000 packets are gathered. The following Fig.1 shows the distributions of PF scheduling delay. Respectively, Fig.1(a) is a histogram of "variable rate with no channel error," and Fig.1(b) is a histogram of "variable rate with 10% channel errors". Also, cumulative lines are drawn in both cases.

[1] Note that one slot takes 1.667 ms.



(a) Variable rate with no channel error



(b) Variable rate with 10% channel errors

Fig. 1. Distribution of packet transmission delay

In "no channel error" case, we get 775 slots with average, which are taken for about 1.29 sec. And, there are above 800 slots for 48.86% of packets. In "10% channel errors" case, average 880 slots are observed for about 1.46 sec. Also, there are above 889 slots for 48.47% of packets.

4 Conclusion

In this paper, we proposed an analytic delay model for PF scheduler. Although the model includes only the scheduling delay and the retransmission delay, the simplification does not undermine the inter-packet interval of a user. In addition, NS-2 simulation result shows that the analytic model approximates to the simulation model.

References

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